Coresets for Farthest Point Problems in Hyperbolic Space*

Eunku Park† Antoine Vigneron‡

Nearest-neighbor and farthest-neighbor searching are fundamental problems in computational geometry. While these problems have been extensively studied in Euclidean space, efficient solutions for non-Euclidean geometries are increasingly important due to their relevance in network embeddings and machine learning. This paper addresses the farthest point problem in a fixed-dimensional hyperbolic space $\mathbb{H}^D$. Specifically, given a set P of n points, the goal is to efficiently identify a point in P that is farthest from a query point q.

The primary contribution of this work is the construction of a coreset (a small, representative subset) for farthest point queries. We demonstrate that for any error tolerance $\varepsilon \in (0, 1)$, it is possible to construct a subset $P_\varepsilon \subset P$ with size bounded by $O(1/\varepsilon^D)$. This construction is highly efficient, requiring only $O(n/\varepsilon^D)$ preprocessing time, which is linear with respect to the input size n.

A distinguishing feature of this coreset is the strength of its approximation guarantees. The constructed subset P_ε satisfies two simultaneous error bounds for any query point q:

1. Multiplicative Error: $d_H(q, p_\varepsilon) \geq (1 - \varepsilon) d_H(q, f_P(q))$.

2. Additive Error: $d_H(q, p_\varepsilon) \geq d_H(q, f_P(q)) - \varepsilon$. Here, d_H represents the hyperbolic metric and $f_P(q)$ is the true farthest point in P.

The construction algorithm adapts to the geometry of the input set P by distinguishing between two cases:

- Small Diameter: When the diameter of P is small (bounded by a constant), the algorithm employs a grid-based bucketing approach similar to techniques used in Euclidean geometry. Points are grouped into buckets based on a grid of mesh size $O(\varepsilon)$, and one representative is kept per bucket.

- Large Diameter: When the diameter is large, simple grid discretization fails due to the exponential growth of volume in hyperbolic space. To resolve this, the authors introduce a cone-based approach. The space is partitioned into cones of angular diameter $O(\sqrt{\varepsilon})$, and the point farthest from the origin within each cone is selected. This is combined with a set of isometric translations to ensure accurate approximations for query points located in various directions.

The proposed coreset has significant algorithmic implications. It allows approximate farthest-point queries to be answered in $O(1/\varepsilon^D)$ time. Furthermore, it yields efficient approximation algorithms for other classic geometric problems in hyperbolic space, including the computation of the diameter, the geometric center, and the maximum spanning tree.

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†Department of Liberal arts and Sciences, DGIST, Daegu, Korea. parkeun9@dgist.ac.kr

‡Department of Computer Science and Engineering, UNIST, Ulsan, Korea. vigneron.antoine@gmail.com